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CRT-2002-10

Centre de recherche sur les transports (C.R.T.)
Centre for Research on Transportation

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April 2002

PUBLICATION
CRT-2002-10

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* This research was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC). The first author was also supported by the Canada Research Chair in Distribution Management. Both authors are affiliated with Center for Research on Transportation, which provided an excellent environment for collaboration. We also like to thank Gilbert Laporte for his helpful comments on an earlier draft of this paper.

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Dépôt légal – Bibliothèque nationale du Québec
Bibliothèque nationale du Canada, 2002

Abstract

This paper presents a model for analyzing a series plant focus strategies in a multicommodity environment. It is based on a facility design model that uses continuous functions in representing the spatial distribution of parameters and the decision variables. The advantage of the model is its ability to generate closed-form solutions with relatively little data. The model allows to study more sophisticated plant focus strategies. Several managerial insights are also discussed.

Key words: location, flexible manufacturing systems, plant focus strategies.

Résumé

Cet article présente un modèle pour l'analyse de plusieurs décisions d'emplacement stratégiques dans un environnement diversifié. Il est basé sur un modèle de localisation qui utilise des fonctions continues pour représenter la distribution spatiale des paramètres de coût et des variables de décision. L'avantage du modèle est sa capacité de produire des solutions compactes avec relativement peu de données. Ce modèle permet l'étude de décisions de localisation stratégiques plus sophistiquées. Plusieurs implications pour les gestionnaires sont également discutées.

Mots-clefs : localisation, technologie flexible, emplacements stratégiques.

1 Introduction

Plant focus involves limiting the number and variety of operations at a factory's production lines in order to increase productivity and lower costs. This concept holds a central place in manufacturing strategy and is used in a variety of forms in different industries. Pharmaceutical firms typically assign products to plants on the basis of sales volumes, whereas auto plants usually focus on manufacturing processes. In consumer appliance industry, firms usually have a "one product-one plant" policy. For example, Maytag has several plants, each dedicated to refrigerators, dishwashers, washers and dryers, or small appliances. In contrast, Dell Computers assembles desktop, laptops, and servers under the same roof using dedicated lines.

The concept of plant focus is not limited to manufacturing industries. Postal services, retailing, and health care systems are among the sectors, in which firms effectively adopt different focus strategies. In retailing, for example, there are specialized stores as well as supermarkets offering wide variety of merchandise. Similarly, hospitals can be specialized in certain clinical services or focused on certain regions. Postal services usually have different pickup, delivery, and sorting centers.

These differences in plant focus result from a series of decisions firms make over time. Product variety, available production technologies, facilities' capabilities and limitations, consumer demographics, and regional variations in operating costs are among the factors that influence those decisions. There has been much conceptual work on the tradeoffs between product, process and market driven focus strategies (Skinner 1974, Hayes and Wheelwright 1984, and Miller and Roth, 1994). Two *pure* focus strategies have received considerable attention in the manufacturing strategy literature (Schmenner 1982):

i. **Product Focus (PF):** Each plant produces only one product to satisfy the demand in its service region. Clearly, there could be several plants producing the same product.

ii. **Market Focus (MF):** Each plant produces *all* of the products to satisfy the demand in its

service region. We assume that each product requires its own facility in the plant.[1]

The purpose of this paper is to develop an analytical framework to analyze some of the tradeoffs involved in plant focus decisions. We consider a firm that is planning to set up a number of plants in order to satisfy the market demand for several products. The firm must decide the number, location, and capacity of the plants as well as their product range and technology content. The service region of each plant also needs to be determined. The tradeoffs involved in these decisions are between the plant opening, operation, and distribution costs.

It is possible to view PF and MF as two endpoints of a wide spectrum of strategies a firm can pursue. Typically, firms adopt a hybrid strategy rather than any of these pure strategies. Nevertheless, the insights gathered by comparing these pure strategies could assist managers in making more refined focus decisions. We also consider a third strategy, which enables us to study the impact of flexible technologies on plant focus decisions:

iii. **Flexible Market Focus (FMF):** Each plant produces all of the products using a flexible technology.

The basis of our framework is a single-commodity facility design model that considers plant opening, capacity acquisition, and distribution costs in the presence of scale economies (Dasci and Verter 2001a). That paper extends and unifies the optimal market area models of Newell (1973), Geoffrion (1979), and Erlenkotter (1989). These classical models focus on the issue of facility size and service region under simplifying assumptions on topology and demand density, leaving determination of the precise locations to a subsequent analysis. This paper makes use of ane optimal market area model in explaining focus strategies and the related tradeoffs. To the best of our knowledge, this is the first such use of optimal market area models. Our analysis led to numerous insights that have not been formalized in the prevailing academic literature.

Our major findings are: i) increasing differences (of any kind) in product characteristics favor prod-

[1]We refer to a production unit dedicated to a product as a *facility* and a collection of co-located facilities as a *plant*.

uct focus strategies all the time, ii) increasing number of products favors market focus strategies, iii) increasing demand favors product focus, and iv) distribution costs could have a substantial impact on the choice of production technology.

The most relevant literature comprises papers that study plant focus decisions in the context of production-distribution system design. Cohen and Moon (1991) present a plant loading problem that is characterized by fixed facility, concave production, and inbound and outbound transportation costs along with scale and scope effects. Their main conclusion is that the outbound transportation costs can have a significant role in reducing plant focus. Billington and Davis (1992) report on a facility design model in Hewlett Packard's (HP) supply chain to decide the set of products assigned to each plant. They describe a discrete facility location model that explicitly includes scale and scope effects in ways to be useful to and used by middle and senior level managers. There are also a number of multi-product location models that can assist focus strategy analysis such as, Warszawski (1973), Klincewicz and Luss (1987), Dasci and Verter (2001b), and Verter and Dasci (2002). However, majority of these authors have not explicitly stated the relevance of their work to plant focus decisions.

Aforementioned papers are based on mixed integer formulations. Even though mixed integer models are capable of identifying the optimal solution, they fail to explain why the optimal solution is what it is. For example, Cohen and Moon (1991) had to undertake a comprehensive parametric analysis to derive results. Furthermore, as the mixed integer models get more complex, data and computational requirements increase considerably, which in turn decreases model stability and applicability. Our results, on the other hand, are mostly simple algebraic expressions that are insightful and intuitive.

There are also a number of papers that analyze plant focus from different perspectives than ours. For example, Benjaafar and Gupta (1998) and Buzacott (1999) use queuing models that incorporate operating parameters and scheduling policies to analyze the impact of production technologies on plant focus decisions. Another stream of research studies the choice between flexible and dedicated technology alternatives (e.g., Fine and Freund 1990, Li and Tirupati 1994 and Van Mieghem 1998).

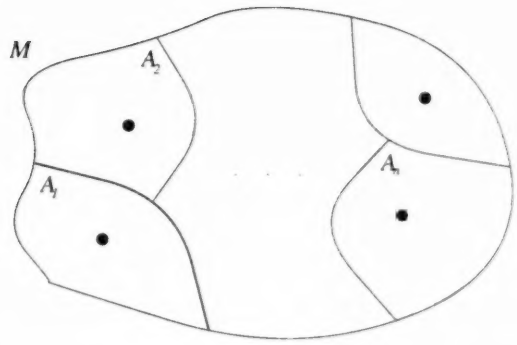

Figure 1: An Example Solution

These papers, however, do not incorporate the location and distribution aspects of plant focus strategies.

The organization of the paper is as follows: Next section is devoted to the development of the basic facility design model. Section 3 presents our multicommodity framework along with an illustrative example. Section 4 is devoted to the analysis of product focus and market focus strategies. Section 5 elaborates on the impact of flexible production technologies and Section 6 concludes the paper with several remarks and future research directions.

2 A Facility Design Problem

In this section, we present a single-product model that constitutes the basis of our analytical framework. The objective is to minimize the total cost of opening and operating a set of plants to serve the customer demand in a given market area. Each plant will serve a subset of the market area, which is called a *service region*. Since all of the market demand is to be satisfied, the service regions have to pack the market area as depicted in Figure 1.

In line with the optimal market area approach, we do not *not* consider the plant location decisions explicitly. Instead, our model determines the service regions leaving precise locations to a subsequent analysis. In addition to the classical optimal market area models, this approach has also

$\mathcal{A}_{ip}$	The service region to be served by facility i,
A_{ip}	Area of the service region i (sq-mile),
$D_p(x,y)$	Demand density at $(x,y) \in \mathcal{M}$ (items/sq-mile),
$F(x,y)$	Fixed cost of opening a plant at $(x,y) \in \mathcal{M}$ (\$),
$f_p(x,y) + b_p(x,y)w$	Capacity acquisition and operating cost of opening a facility of size w at $(x,y) \in \mathcal{M}$ (\$),
$c_p(x,y)$	Freight rate of shipments originating from (x,y) (in \$/item-mile),

Table 1: Decision Variables and Parameters for Product p

been used by Webster and Gupta (1995) and Erlebacher and Meller (2000).

Table 1 presents the notation. We consider two kinds of fixed costs: $F(x,y)$ includes only the fixed costs associated with opening plants, irrespective of the number and size of the facilities, whereas $f_p(x,y)$ represents the fixed costs associated with acquiring facility to produce product p. Although this distinction is a mute point in a single commodity environment, the interplay between these fixed costs has important implications in the analysis of focus strategies.

For the ease of exposition, we will omit subscript p and coordinates (x,y) throughout the remainder of this section. The single product facility design problem can be formulated as follows:

$$\min_{A_s} [\text{TC}] = \int_{\mathcal{M}} [\frac{F}{A_s} + \frac{f}{A_s} + bD + cKD\sqrt{A_s}]dxdy,$$

where $A_s(x,y)$ is a step function defined over two-dimensional market area as $A_s(x,y) = A_i$ for $(x,y) \in \mathcal{A}_i$ and $K(x,y)$ is a constant. Note that, the first term, $F(x,y)/A_s(x,y)$, is nothing but the "average" fixed cost per square mile when a plant is located at $(x,y) \in A_i$. If the fixed cost is slowly varying in a service region, this average could be a quite accurate estimation of the actual fixed cost per square mile incurred by the firm. The same argument applies to the fixed capacity cost, $f(x,y)/A_s(x,y)$.

The transportation cost depends on the shape of the region, location of the facility in the service region, distance metric, vehicle sizes, and the shipment schedules. Here, the transportation system is of one-to-many type, which has been extensively studied in the logistics literature (e.g., Burns et al. 1985, Daganzo 1991, Rosenfield, Engelstein, and Feigenbaum 1992, and Hall 1993). This literature mostly deals with sizing of vehicle loads, vehicle service regions and shipment schedules

to minimize the total transportation and inventory costs under a variety of assumptions.

In order to simplify the analysis, we assume that the unit transportation cost is proportional to the distance traveled. It is possible to express the total transportation cost as such in many real life logistics systems. Examples include the cases with direct full truck load shipments and the cases where the vehicles visit many customers at a dispatch[2]. It is common to approximate the average distance from a plant to the customers by $K(x,y)\sqrt{A_s(x,y)}$, where $K(x,y)$ is a constant that depends on the distance metric (road structure) and the shape of the service region. Erlenkotter (1989) points out that, K is relatively robust to the distance metric and the shape of the service region.

The characterization of $A_s(x,y)$ is not an easy task. Therefore, we first find an optimal smooth function, $A(x,y)$, (i.e., a *continuous approximate* to $A_s(x,y)$) then find a step function that is consistent with the smooth function. This procedure is known as continuous approximation, which is presented in detail in Newell (1973). Therefore, the problem can be reformulated as follows:

$$\min_{A} \ [\mathrm{TC}] \ = \ \int_{\mathcal{M}} [\frac{F+f}{A} + bD + cKD\sqrt{A}]dxdy. \tag{1}$$

A smooth function that minimizes integral (1), also minimizes the integrand at each point in the market area. Therefore, we need to solve the following problem for each $(x,y) \in \mathcal{M}$:

$$\min_{A} \ \overline{\mathrm{TC}} = [\frac{F+f}{A} + bD + cKD\sqrt{A}].$$

It is easy to show that $\overline{\mathrm{TC}}$ is strictly quasiconvex, hence, the first order condition suggests:

$$A^* = \left(\frac{2(F+f)}{cKD}\right)^{2/3}, \tag{2}$$

and after some simple algebra,

$$\mathrm{TC}(A^*) = \int_{\mathcal{M}} bDdxdy + \int_{M} \frac{3}{\sqrt[3]{4}}(F+f)^{1/3}(cKD)^{2/3}dxdy \tag{3}$$

Similar results are obtained by Newell (1973), Geoffrion (1979), and Erlenkotter (1989) when the demand density and cost parameters are constant over the market area. In their cases, A^* is a constant whereas in our case it is a surface defined over two-dimensional plane.

[2]An extensive discussion can be found in Burns et al. (1985), Daganzo (1991), and Dasci (2001).

The optimal cost, TC(A^*), is insensitive to small errors in A^* or model parameters. This robustness property is important also because it is an indication that the geometry of service regions are not crucial for the model. The shapes of the service regions and the road structure influence the solution only through K values, and as noted earlier, K itself is insensitive to them.

Note also that, the parameters are defined as continuous functions over the market area. It could be argued that, these definitions imply collection of large amount of data. However, we do not require that those parameters should be estimated at every point in the market area. Companies usually have "ballpark" figures on these parameters prior to the establishment of actual facilities. Several regional macroeconomic indicators could be used to adjust these ballpark figures to accommodate geographical differences. Real estate rates, wage rates, subsidies, population demographics, etc. are among such regional information that are estimated and reported periodically by several public and private institutions. Such an estimation method is less time consuming and inexpensive. It is also quite accurate because the solution is insensitive to small errors in these parameters.

We conclude this section by pointing out that, although the facility design model is developed as a basis for our comparative analysis, it can also be used to determine the actual service regions. We present a detailed procedure for such an application in Appendix A.

3 The Analytical Framework

In this section, we elaborate on the three plant focus strategies, PF, MF, and FMF. First, based on the results presented in the previous section, we determine the optimal solution for each strategy. Then, we solve an illustrative example.

3.1 Focus Strategies

To simplify the analysis we assume that $c \equiv c_p$ for all products. This assumption is not restrictive, because it is possible to scale the demand and cost data (D_p's and b_p's) accordingly. Therefore, in the remainder of the paper, D_p and b_p represent the *scaled* demand densities and capacity costs,

respectively. Let us rewrite (2) and (3) for product p:

$$A_p^* = \left(\frac{2(F+f_p)}{cKD_p}\right)^{2/3}, \tag{4}$$

$$\mathrm{TC}^*(f_p,\ b_p,\ D_p) = \int_{\mathcal{M}} b_p D_p dxdy + \int_{\mathcal{M}} \frac{3}{\sqrt[3]{4}}(F+f_p)^{1/3}(cKD_p)^{2/3}dxdy. \tag{5}$$

Parameterization of the optimal cost, as in (5), allows us to better express the optimal total cost under each focus strategy. Recall that, facility design decision for each product are made separately under PF. Therefore,

$$\mathrm{TC[PF]} = \sum_p \mathrm{TC}\ (f_p,\ b_p,\ D_p). \tag{6}$$

Under MF, each plant produces all the products demanded in its service region. Therefore, it is possible to model MF using a single *pseudo-product*, for which the demand density is the sum of all product demand densities ($\sum_p D_p$). The fixed capacity cost is also simply the sum of all fixed capacity costs ($\sum_p f_p$). The unit capacity cost of the pseudo product is the weighted average of all unit capacity costs, where the weights are the corresponding demand densities, i.e.,

$$W(b_p, D_p) = \sum_p \frac{b_p D_p}{\sum_p D_p}.$$

Therefore, optimal service region and total cost of the MF strategy could be expressed as:

$$A^* = \left(\frac{2(F+\sum f_p)}{cK\sum D_p}\right)^{2/3}, \tag{7}$$

$$\mathrm{TC[MF]} = \mathrm{TC}^*(\sum_p f_p,\ , W(b_p, D_p),\ \sum_p D_p). \tag{8}$$

Under FMF, there is one flexible facility at each plant with a fixed capacity cost of f_0 and unit capacity cost of b_0. Without loss of generality, we assume that variable capacity cost, b_0, is the same for all products. We do however, consistent with technology selection literature, require that b_0 is greater then all b_p's. Hence,

$$A^* = \left(\frac{2(F+f_0)}{cK\sum D_p}\right)^{2/3}, \tag{9}$$

$$\mathrm{TC[FMF]} = \mathrm{TC}^*(f_0,\ b_0,\ \sum_p D_p). \tag{10}$$

Our purpose is to derive some managerial insights with regards to these plant focus strategies. At their current form they are not amenable to such analyses and further specifications are needed. Before turning to the comparative analysis, however, we provide an illustrative example.

3.2 An Illustrative Example

We present a two-product example in order to highlight the differences among PF, MF, and FMF. It should be taken as illustrative rather than suggestive. Let $\mathcal{M}$ be a square service region with sides of 100 miles. The following parameters are defined at each $(x, y) \in \mathcal{M}$:

$$F = 5000, \quad f_1 = f_2 = 1000, \quad f_0 = 4000, \quad c = 1, \quad K = 0.38,$$
$$D_1(x, y) = \frac{200 + 9x + 9y}{2200}, \text{ and } D_2(x, y) = \frac{2000 - 9x - 9y}{2200},$$

Note that, all the parameters are constant across $\mathcal{M}$ except the demand. Demand densities are linear in both x and y, and their maximum is 10 times their minimum (i.e., $D_1(0,0) = D_2(100,100) = 1/11$ as compared to $D_1(100,100) = D_2(0,0) = 10/11$). Unit capacity costs ($b_0, b_1$, and b_2) are not specified because they have no effect on the optimal market areas. Note also that, there is no economies of scope in flexible technology acquisition since $f_0 > f_1 + f_2$. Using (4) we compute the following optimal market areas under PF:

$$A_1^*(x, y) = \frac{169,000}{(200 + 9x + 9y)^{2/3}} \quad \text{and} \quad A_2^*(x, y) = \frac{169,000}{(2000 - 9x - 9y)^{2/3}}$$

$A_1^*(x, y)$ is a decreasing function in both x and y with a minimum of $A_1^*(100, 100) = 1065$ and a maximum of $A_1^*(0, 0) = 4940$ ($A_2^*(x, y)$ is nothing but a mirror image of $A_1^*(x, y)$). Despite the tenfold difference between demand densities across the market area, the largest possible service region is only five times bigger than the smallest.

The average service region sizes are $\bar{A}_1^* = \bar{A}_2^* = 1717$, which suggest that there will be five or six service regions ($10,000/1717 = 5.82$). Using the procedure outlined in Appendix A we determine the service regions for product 1 and product 2 under PF, which are depicted in Figure 2 (top left and right images respectively). It is interesting that, the resulting largest service region is in fact less than twice the smallest.

We use (7) to determine $A^* = 1107$ under MF, which suggests nine plants. Note that $D_1(x, y) + D_2(x, y) = 1$ and hence the total demand density is constant across the market area. This does not necessarily make the task of identifying the service regions any easier. Therefore, we again utilized the primitive descent algorithm outlined in Appendix A. Figure 2 (bottom left) shows the

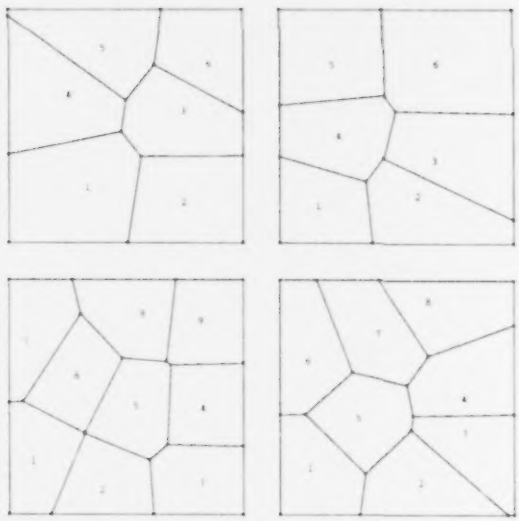

Figure 2: Service Regions under PF (top), MF (bottom left), and FMF Strategies

service regions under MF. Using (9) we determine $A^* = 1309$ under FMF, suggesting eight plants. The resulting service regions are shown in Figure 2 (bottom right)[3].

In this illustrative example adopting PF involves more plants than MF (6+6=12 vs. nine). In the next section we prove that this result is valid in general. Given that there are two facilities at each MF plant, MF involves more facilities than PF (18 vs. 12). This result, however, does not hold in general. Although the difference between maximum and minimum demand densities is tenfold in this example, the final service regions are only two times different. That is, our solution methodology smoothes out large differences across the market area.

4 Product Focus vs. Market Focus

In this section, we analyze the impact of problem parameters on the choice between PF and MF strategies. We simplify the exposition of the material by assuming that all problem parameters are

[3]The final service region shapes depend on the initial Voronoi points and the parameters of the algorithm. Therefore, a slightly different solution can be reached if one of the procedure parameters or the starting solution is modified.

constant across the market area. However, this assumption is not restrictive from a methodological perspective because it is quite straightforward to derive similar results and insigts when the parameters are slowly varying. First, let us formalize one of the results observed in our illustrative example:

Proposition 1 *PF holds more plants than MF.*

The proof is provided in Appendix B. The choice between the two strategies will be made on the basis of total cost. Through detailing expressions (6) and (8), the optimal total costs (per sq-mile) under each strategy can be expressed as follows:

$$\text{TC[PF]} = \sum_p \left[b_p D_p + \frac{3}{\sqrt[3]{4}} (F + f_p)^{1/3} (cD_p)^{2/3} \right], \text{ and} \tag{11}$$

$$\text{TC[MF]} = \sum_p b_p D_p + \frac{3}{\sqrt[3]{4}} (F + \sum_p f_p)^{1/3} \left(\sum_p cD_p \right)^{2/3}. \tag{12}$$

It is clear that the strategy with the lower optimal cost should be preferred.[4] Hence,

Observation 1 *MF is preferred to PF if and only if*

$$\sum_p \left(\frac{D_p}{\sum_p D_p} \right)^{2/3} (F + f_p)^{1/3} \geq (F + \sum_p f_p)^{1/3}$$

The proof is straightforward by regrouping the components of (11) and (12)[5].

Apparently the shipment rate, c, has no direct impact on the choice. This is due to the scaling done in the previous section. This implies that in cases where shipment rates of different products are indeed equal (as in auto transportation where each truck carries eight mid-size cars) the shipment rates are irrelevant to the decision.

In general, the disadvantage of PF over MF is that, the fixed cost, F, is incurred for each plant. Proposition 1 implies that the total fixed cost under PF is higher. When the fixed cost is small,

[4] We assume that the firm prefers MF also when the total costs are equal.

[5] We call such findings as *observation* because their proofs are no more involved than regrouping the terms

however, this disadvantage of PF becomes insignificant. The following proposition delineates the choice for $F = 0$:

Proposition 2 *PF is preferred to MF if* $F = 0$.

The proof is based on Hölder's Inequality and is provided in Appendix B. Above condition is sufficient but not necessary. Proposition 3 shows that PF can actually be preferred for positive values of F. Let r denote the number of products. The proof of the following proposition is based on Minkowski's inequality and provided in Appendix B.

Proposition 3 *When* $D_p = D$ *for all* p,

a) MF is preferred to PF if

$$F \geq \frac{r}{r-1}\left(M_1(f) - M_{1/3}(f)\right)$$

b) PF is preferred to MF if

$$F < \left(\frac{\sqrt{(r^2-r)M_1(f) + rM_{1/2}(f)} - r\sqrt{M_{1/2}(f)}}{r-1}\right)^2$$

where $M_t(f) = \left(\frac{\sum_{p=1}^{r} f_p^t}{r}\right)^{1/t}$ and is known as *mean of order* t *of* f_p (see Beckenbach and Bellman 1961)[6]. Most of the results in this paper are expressed in terms of *fractional* means (i.e., $M_t(f)$, $0 < t < 1$). We have observed that a fractional mean can be considered as a surrogate measure for the variability in the underlying data set. As the variability of data increases, fractional means *decrease* and in general, the smaller the order t, the larger the rate of decrease. If there is no variability, all fractional means reach to their upper bound, i.e., $M_1(f)$.

Proposition 3a suggests that for sufficiently large F, MF is always preferred to PF. However, as the variability (and/or magnitude) of f_p's increase PF becomes more attractive. Similarly, Proposition

[6]For example, $M_1(f)$ and $M_0(f)$ are known as arithmetic and geometric means respectively. It is easy to see that $M_t(f) = M_1(f)$ for all t if all f_p's are equal. In general, $M_t(f)$ is nondecreasing in t, such that

$$\min\ f_p \leq M_t(f) \leq \max\ f_p \text{ for } -\infty < t < \infty.$$

3b enables us to compute the upper bound on F, below which PF is always preferred. The analysis is inconclusive when neither condition is satisfied. Proposition 3b also implies that when capacity fixed costs, f_p, are considerably higher than plant fixed cost, F, PF will be preferred.

It is possible that the plant fixed cost is considerably higher under MF than PF. Because MF plants contain r facilities as opposed to the "one plant one facility" structure under PF. If this is the case we can revise Proposition 3a as:

a') $F - F_{MF}(r)/r \geq M_1(f) - M_{1/3}(f)$,

where $F_{MF}(r)$ denotes the fixed cost of a plant under MF as a function of the number of products. If the number of products is very large (i.e., as $r \to \infty$), then the conditions become:

a") $F \geq M_1(f) - M_{1/3}(f)$,

b") $F < \left(\sqrt{M_1(f)} - \sqrt{M_{1/2}(f)}\right)^2$.

Now we turn to an analysis for the cases where fixed capacity costs are equal for all products. In this analysis, our focus is on the impact of demand variability among products.

Proposition 4 *When $f_p = f$, MF is preferred to PF if and only if*

$$\frac{F/r + f}{F + f} \leq \left(\frac{M_{2/3}(D)}{M_1(D)}\right)^2. \tag{13}$$

Clearly, both sides of the inequality are less than one. The right hand side is maximized when all D_p's are equal (i.e., $M_{2/3}(D)/M_1(D) = 1$) in which case, MF is preferred to PF. Note that this result can also be observed from Proposition 3a. The right hand side is minimized when $D_p = 0$ for all but one product. Then, the right hand side becomes $1/r$ and hence, PF is preferred to MF. These two extreme cases show that the variability in (scaled) demand densities plays an important role in choosing a focus strategy[7].

[7]The variability as used here, is not the variability of demand densities over the market area. It refers to the variability among products' demand densities, as the demand density of each product is still constant over market area.

In closing this section, we provide further remarks on our findings:

i. **As the differences in product characteristics increase, PF becomes more favorable.** Suppose that one product is considerably different from the others. This product does not influence the plant configurations of the others under PF because each product is treated separately. However, all products' configurations are interdependent under MF and hence, the resulting service regions could become far from optimal. Therefore, as the products become quite different in terms of their costs and demands PF would be more preferable.

ii. **As the number of products increases, MF becomes more favorable.** Under PF, new plants are to be opened and new production units are to be acquired each time an additional product is introduced. Under MF, however, the establishment of new plants is not necessary. Hence, increasing number of products favors MF as long as $F_{MF}(r)/r$ is decreasing.

iii. **Shipment rates alter the strategy choice through their impact on scaled demand densities.** As noted earlier, if the shipment rates are actually equal, they do not have any impact on focus decisions. Similarly, if the shipment rates and/or demand densities change for all products at the same rate then focus decisions will also be unaffected by these changes.

    It does not, however, mean that focus strategies are independent of transportation costs. In general, the impact of shipment rates should be considered in conjunction with the scaled demand densities. If increasing differences in shipment rates bring the scaled demand densities closer, then MF is favored. Otherwise, PF is favored.

5 The Impact of Flexible Technology

In this section we analyze the impact of flexible production technology that is able to produce all products. As noted earlier we assume that unit capacity cost of flexible technology is higher than those of dedicated technologies (i.e., $b_0 > b_p$ for all p). We first compare FMF to MF and then to PF. We also present an analysis of the impact of regional differences on plant focus. Using (10),

the optimal cost per sq-mile can be written as:

$$\text{TC[FMF]} = b_0 \sum_p D_p + \frac{3}{\sqrt[3]{4}}(F + f_0)^{1/3} \left(\sum_p cD_p \right)^{2/3}. \quad (14)$$

5.1 FMF vs. MF

The conventional paradigm in the literature is that the logistics factors have no impact on the production technology selection decisions. Although the plants under FMF and MF strategies have similar roles in serving the regional markets, we will show that this convention is not necessarily valid.

Observation 2 *MF is preferred to FMF if and only if*

$$\left(F + \sum_p f_p \right)^{1/3} - (F + f_0)^{1/3} \leq \frac{W(b_0 - b_p, D_p)}{\tilde{c}} \left(\sum_p D_p \right)^{1/3}$$

where

$$W(b_0 - b_p, D_p) = \sum_p \frac{D_p}{\sum_p D_p}(b_0 - b_p) \text{ and } \tilde{c} = 3c^{2/3}/\sqrt[3]{4}$$

Note that $b_0 > b_p$ for all p implies $W(b_0 - b_p, D_p) > 0$, and hence the right hand side is positive. In this case, FMF may be preferable only if the flexible technology provides economies of scope in fixed capacity costs. In other words the flexible technology is never preferred if $f_0 \geq \sum_p f_p$. In cases where $f_0 < \sum_p f_p$, the attractiveness of FMF depends on the other parameters. For example, higher shipment rates, $\tilde{c}$, would favor FMF, whereas higher demand densities, D_p, would favor MF.

Capacity acquisition costs have long been considered as the main factor in choosing between product-dedicated and flexible technologies. Our analysis, however, points out that capacity cost is not the only factor that determines the preferable technology. In the technology selection literature, the ability of flexible technologies to hedge against demand uncertainty is considered as their prime advantage (Fine and Freund 199 and Van Mieghem 1998). We also show that flexible technologies can be preferred even under deterministic demand.

5.2 FMF vs. PF

In the remainder of this section, let $F_p = F + f_p$ and $\bar{b}_p = b_0 - b_p$ for all $p = 1, \ldots, r$ and $F_0 = F + f_0$. The following three observations are the counterparts of Observation 1 and Propositions 3 and 4.

Observation 3 *PF is preferred to FMF if and only if*

$$\sum_p F_p^{1/3} D_p^{2/3} - F_0^{1/3} \left(\sum_p D_p \right)^{2/3} < \frac{1}{\tilde{c}} \sum_p \bar{b}_p D_p \tag{15}$$

Observation 4 *When $D_p = D$, PF is preferred to FMF if and only if*

$$[M_{1/3}(F_p)]^{1/3} - \left(\frac{F_0}{r} \right)^{1/3} < \frac{M_1(\bar{b}_p)}{\tilde{c}} D^{1/3} \tag{16}$$

Observation 5 *When $F_p = \bar{F}$ and $\bar{b}_p = \bar{b}$ for all $p = 1, \ldots, r$, PF is preferred to FMF if and only if*

$$\bar{F}^{1/3} \left(\frac{M_{2/3}(D_p)}{M_1(D_p)} \right)^{2/3} - \left(\frac{F_0}{r} \right)^{1/3} < \frac{\bar{b}}{\tilde{c}} [M_1(D_p)]^{1/3} \tag{17}$$

All of these observations could easily be proven by comparing (11) and (14). The following remarks summarize the insights that are gathered from these observations:

i. **As the differences in product characteristics increase, PF becomes more favorable.** The left hand sides of (16) and (17) decrease as the differences in fixed costs and demand densities increase.

ii. **As the number of products increases, FMF becomes more favorable.** Clearly, F_0/r decreases as r increases, this in turn, increases the left hand sides of (16) and (17).

iii. **As the shipment rates increase, FMF becomes more favorable.** The right hand sides of (15), (16), and (17) decrease as $\tilde{c}$ increases. In contrast with the results presented in the previous section shipment rates have a direct impact on the choice between FMF and PF.

iv. **Increase in demand densities favors PF.** As demand (D_p's) increases, left hand sides of (15), (16), and (17) increase faster than their right hand sides.

v. **As the flexible technology becomes more expensive PF becomes more favorable.** This result is quite intuitive and actually does not require an analysis such as above. However, it is indication that our analytical framework is accurate and insightful. Flexible technology could be more expensive either through fixed cost (F_0) or unit capacity cost (b_0). In the former case, left hand sides of the inequalities decrease, whereas in the latter case right hand sides increase; both favoring PF over FMF.

5.3 Impact of Regional Differences

Our general framework allows comparison of the strategies when there are regional differences. Intuitively, all geographic differences should favor PF over FMF. For example, if consumer preferences are dramatically different in the east and west coasts, it could be favorable to setup plants with different product ranges to serve their stronger markets more efficiently. Similar reasoning applies if there are geographical differences in the factor costs.

There are variety of combinations we could analyze. But considering the similarity of each analysis, we restrict ourselves to a case where only the demand density varies over the market area. Furthermore, we also assume that cost parameters are not only constant across the market area but also equal for all products (i.e., $F_p = F$, $b_p = b$ for all $p = 1, 2, \ldots, r$). This assumption is not restrictive for the analysis but, it helps us to highlight the impact of geographical differences. The proof of the following lemma is provided in Appendix B.

Lemma 1 *PF is preferred to FMF if and only if*

$$\Psi_1 F^{1/3} - \Psi_2 (\frac{F_0}{r})^{1/3} < \frac{\bar{b}}{\bar{c}}$$

where

$$\Psi_1 = \frac{\int_{\mathcal{M}} \left(M_{2/3}[D(x,y)] \right)^{2/3} dxdy}{(\int_{\mathcal{M}} M_1[D(x,y)] dxdy)}, \text{ and } \Psi_2 = \frac{\int_{\mathcal{M}} (M_1[D(x,y)])^{2/3} dxdy}{(\int_{\mathcal{M}} M_1[D(x,y)] dxdy)}$$

and

$$M_t[D(x,y)] = \left(\frac{\sum_{p=1}^{r} D_p^t(x,y)}{r} \right)^{1/t}$$

Observation 6 *Increasing regional differences favor PF over FMF.*

As the geographical variation increases the numerator of Ψ_1 would decrease while that of Ψ_2 stays constant (see Beckenbach and Bellman 1961, pp. 16,17). As a result, the left hand side decreases and hence, PF becomes more favorable. It could be shown that, this result is not limited to the demand but valid for all parameters. The following example is devised to gather more insights into the impact of regional differences.

As an illustrative example, we consider two products that are identical in terms of all the pertinent costs. In the following two parts we will make different assumptions on their demand densities to highlight the impact of different forms of geographical variations.

a) Let us assume that both products have the same demand density as:

$$D_p(x,y) = \left(\frac{2}{\lambda+1} + \frac{2(\lambda-1)x}{(\lambda+1)\sqrt{M}}\right)\bar{D} \quad \forall (x,y) \in \mathcal{M},$$

where $\bar{D}$ is a constant. Clearly, both of the demands are increasing on the x-axis, but constant in the other. Note that, λ is the ratio of the highest demand density to the lowest demand density in the market area. Here, λ serves as a measure of degree of regional differences. If we apply Lemma 1 and evaluate the integrals therein we reach the following:

PF is preferred to FMF if and only if

$$\Psi(\lambda)\left(F^{1/3} - (\frac{F_0}{2})^{1/3}\right) < \frac{\bar{b}}{\bar{c}}\bar{D}^{1/3}$$

where $\Psi(\lambda) = \frac{3\sqrt[3]{4}}{5}(\frac{\lambda^{5/3}-1}{(\lambda+1)^{2/3}(\lambda-1)})$. $\Psi(\lambda)$ is strictly decreasing and $\Psi(\lambda) \to 1$ as $\lambda \to 1$ and $\Psi(\lambda) \to 0.849$ as $\lambda \to \infty$. We can observe that as the geographical differences increase (i.e., λ increases) left hand side decreases and hence, PF would be more favorable. It could easily be verified that when there are no regional differences ($\lambda = 1$) the result is equivalent to that of Observation 5.

b) In this part we assume that product demands are mirror image of each other's (geographically). $D_1(x,y)$ is as before but, $D_2(x,y) = 2\bar{D} - D_1(x,y)$, with the same average demand. In this case,

PF is preferred to FMF if and only if

$$\Psi(\lambda)F^{1/3} - \left(\frac{F_0}{2}\right)^{1/3} < \frac{\bar{b}}{\bar{c}}D^{1/3}$$

where $\Psi(\lambda)$ is same as in Part (a). Similarly increasing geographical differences favors PF over FMF. However, more importantly, increasing λ favors PF even *more* than Part (a). This result might not seem intuitive at first, because in this case flexible technology is facing an ideal situation of constant total demand density all across the market area. However, considering the cases where PF has been favored, this is another instance where differences in product characteristics favor PF. What is different here is the geographical demand dispersion.

6 Concluding Remarks

In this paper, we present a continuous approximation framework for studying the plant focus decisions in a manufacturing environment. The proposed framework, however, is equally applicable to those service industries where proximity of facilities and to customers is an important issue. One of the differentiating features of our methodology is that it needs relatively little data. For the most part, our analysis requires only some fractional means of the model parameters. In contrast, the mixed integer models proposed in the literature have extensive data requirements. Also, their explanatory capability with regards to the choice among focus strategies is quite limited. This paper constitutes a much-needed effort to remedy these weaknesses of the prevailing mixed integer formulations. Nevertheless, it is important that continuous approximation and integer programming approaches do not compete but complement each other. The former provides general strategic insights about plant focus, whereas the strength of the latter is its ability to prescribe accurate solutions.

The results of this paper can be extended in several directions. First, incorporation of uncertainty constitutes a fruitful avenue for future research. Unlike the case of mixed integer formulations, including uncertainty into our framework is relatively easier, because the solution of our model has a simple structure. Our results resemble EOQ models, and therefore the stochastic inventory literature can provide a source of inspiration here.

The second extension is incorporation of detailed logistical factors in our framework. A more accurate representation of transportation and inventory costs may have a significant impact on the nature of the solutions. Such a study would be quite helpful to understand the impact of tactical decisions on plant focus decisions. Finally, our framework can be utilized for designing focus strategies rather than evaluating pre-conceived strategy alternatives. Adaptation of the "power of two" policies, which are well known in inventory control, looks promising.

Appendix A: Identifying the Service Regions

As pointed out earlier, there is a one-to-one relationship between the collection of service regions defined over a market area and an appropriate step function. Therefore, if we could find a step function that is consistent with the smooth approximate (A^*) we could also gather all the information pertaining to the service regions. Here, we present an algorithm to find an appropriate step function. Several objective functions are possible to this end. We choose to minimize the sum of absolute differences between the smooth and the step function which will be called the total *inconsistency*. The problem could be formulated as follows:

$$\min_{A_s(x,y)} S = \int_{\mathcal{M}} |A_s(x,y) - A^*(x,y)| dxdy. \tag{18}$$

To implement it successfully, certain policies regarding the choice of initial solution and stopping criterion should be specified. Note however that, we really do not need to find the global optimal solution, not because it is a difficult problem itself but because a good solution will serve the purpose of continuous approximation, as we are concerned only with a *consistent* step function.

Let $\bar{A}^*$ be the average value of $A^*(x,y)$ and $m = M/\bar{A}^*$ be the expected number of facilities. Even in the unlikely event that m is an integer, we should try several integers close to m and choose the best. Let n denote this integer. We start with n points (called generators) on the plane and generate the bounded Voronoi diagram. Each Voronoi polygon can be interpreted as a service region and each segment of the step function could be defined on these polygons. Hence, the objective function (18) depends only on the generators after this transformation. Let

Step 0:	Start with initial $\boldsymbol{x}^{(0)}$.
Step 1:	Construct the Voronoi diagram of $\boldsymbol{x}^{(m)}$, find $A_s(\boldsymbol{x}^{(m)})$.
Step 2:	Determine the search direction $\boldsymbol{d}^{(m)} = -\nabla S(\boldsymbol{x}^{(m)})$
Step 3:	Solve the following to obtain $\hat{\lambda}$
	$S(\boldsymbol{x}^{(m)} + \hat{\lambda}\boldsymbol{d}^{(m)}) = \min_{\lambda} S(\boldsymbol{x}^{(m)} + \lambda \boldsymbol{d}^{(m)})$
Step 4:	Update the solution $\boldsymbol{x}^{(m+1)} = \boldsymbol{x}^{(m)} + \hat{\lambda}\boldsymbol{d}^{(m)}$
Step 5:	Stop if $\lvert\boldsymbol{x}^{(m+1)} - \boldsymbol{x}^{(m)}\rvert \leq \epsilon$, otherwise go to Step 1.

Table 2: The primitive descent algorithm

$\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n}$ be the generator points and $V_1, V_2, \ldots, V_n$ be the corresponding Voronoi polygons. Then,

$$A_s(x, y) = |V_i|, \text{ for } (x, y) \in V_i$$

where $|V_i|$ denotes the area of the ith polygon. Hence, the problem (18) could be expressed as:

$$\min_{\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n}} S = \sum_{i=1}^{n} \int_{V_i} ||V_i| - A(x, y)| dx dy. \tag{19}$$

The plant locations are not necessarily the generator points and can be determined by a subsequent analysis. It suffices to say that, plant locations could be found by solving a generalized Weber problem for each service region or several alternative central locations could be compared and best among them could be chosen.

The objective function (19) is non-convex and non-differentiable in general. Therefore, we modify the primitive descent algorithm proposed in Suzuki, Asami and Okabe (1991) for a different problem. The algorithm starts with n points and the objective function and the primitive descent direction is computed. Then, the points are updated at that direction with best step size, which is computed by a line search. It is terminated, when the l_∞ distance of successive solutions are less than a threshold. Although, the algorithm stops at a local optimum, this is not a serious drawback because it produces a *consistent* step function. Table 6 summarizes the algorithm.

There are certain policies regarding the choice of initial solution and stopping criterion to be set in order to implement the algorithm successfully. A good initial solution could be chosen by inspection if n is small. However, we coded a subroutine (which is more likely to be more useful for larger n)

to find a reasonably good starting solution. Our subroutine is based on the successive generation and selection of pseudo-random points on the market area. Each randomly drawn point is tested to find if it is reasonably far apart from points selected before and from the sides of the market region. It could be best described as if we are trying to locate circular disks with areas equal to the value of $A^*(x, y)$ at those random points such that the discs are (reasonably) non-overlapping. We generate such random points until we reach several starting solutions. Then we choose the best among them and continue from Step 1.

Above algorithm could be refined further for a real life application. But, we were satisfied with its solution for the instances we have tested. It converged to a solution all the time, which is the most important result for the purpose of this study. For a more detailed account on the performance of this algorithm (on a different problem), Suzuki, Asami and Okabe (1991) constitutes an excellent source.

Appendix B

Proof of Proposition 1: It suffices to show that

$$\sum_p \frac{M}{A_p^*} \geq \frac{M}{A^*}$$

which implies:

$$\sum_p \left(\frac{cK D_p}{2(F + f_p)} \right)^{2/3} \geq \left(\frac{cK \sum_p D_p}{2(F + \sum_p f_p)} \right)^{2/3}$$

and

$$\left(\frac{(F + \sum_p f_p)}{(F + f_p)} \right)^{2/3} \sum_p \left(\frac{D_p}{\sum_p D_p} \right)^{2/3} \geq 1$$

The first term is always greater than one. Hence, above inequality is implied by

$$\sum_p \left(\frac{D_p}{\sum_p D_p} \right)^{2/3} \geq 1,$$

which is straightforward.

Proof of Proposition 2: For $F = 0$;

$$\sum_p \left(\frac{D_p}{\sum_p D_p} \right)^{2/3} (F + f_p)^{1/3} = \sum_p \left(\frac{D_p}{\sum_p D_p} \right)^{2/3} (f_p)^{1/3} \quad \text{(because } F = 0\text{)}$$

$$\leq \left(\sum_p \frac{D_p}{\sum_p D_p}\right)^{2/3} (\sum_p f_p)^{1/3} \text{ (Hölder's Inequality)}$$
$$= (\sum_p f_p)^{1/3}$$
$$= (F + \sum_p f_p)^{1/3} \text{ (because } F = 0).$$

Proof of Proposition 3:

a) Note that,

$$\sum_p (\frac{D_p}{\sum_p D_p})^{2/3}(F + \sum_p f_p)^{1/3} = \frac{1}{r^{2/3}}\left[\sum_p (F + \sum_p f_p)^{1/3}\right] \text{ (because } D_p = D)$$
$$\geq \frac{1}{r^{2/3}}\left[(\sum_p F^{1/3})^3 + (\sum_p f_p^{1/3})^3\right]^{1/3} \text{ (Minkowski's Inequality)}$$
$$= \frac{1}{r^{2/3}}\left[r^3 F + (\sum_p f_p^{1/3})^3\right]^{1/3}$$

On the other hand if $(1 - 1/r)F \geq M_1(f) - M_{1/3}(f)$ then

$$(1 - \frac{1}{r})F \geq \frac{\sum f_p}{r} - \left(\frac{\sum f_p^{1/3}}{r}\right)^3, \text{ then}$$
$$r^3 F + \left(\sum f_p^{1/3}\right)^3 \geq r^2(F + \sum f_p), \text{ and}$$
$$\frac{1}{r^{2/3}}\left[r^3 F + \left(\sum_p f_p^{1/3}\right)^3\right]^{1/3} \geq (F + \sum f_p)^{1/3}.$$

b)We will try to find F^* such that for $0 \leq F \leq F^*$,

$$\left[\sum_p \left(\frac{D_p}{\sum D_p}\right)^{2/3} (F + f_p)^{1/3}\right]^3 \leq (F + \sum f_p)$$

Consider,

$$[\sum_p (\frac{D_p}{\sum D_p})^{2/3}(F + f_p)^{1/3} = r(\frac{1}{r}\sum (F + f_p)^{1/3})^3, \text{ (because } D_p = D)$$
$$\leq r\left(\frac{1}{r}\sum (F + f_p)^{1/2}\right)^2, \text{ (because } M_{1/3}(.) \leq M_{1/2}(.)),$$
$$\leq r\left[\frac{1}{r}\left(\sum F^{1/2} + \sum f_p^{1/2}\right)\right]^2, \text{ (because } (F + f_p)^{1/2} \leq F^{1/2} + f_P^{1/2}),$$
$$= r\left(F^{1/2} + \sum \frac{1}{r} f_p^{1/2}\right)^2$$
$$= r\left(F^{1/2} + (M_{1/2}(f))^{1/2}\right)^2$$
$$= rF + 2rF^{1/2}(M_{1/2}(f))^{1/2} + rM_{1/2}(f) \qquad (20)$$

Let us try to find F^* where (20) is less than $F+\sum_p f_p = F+rM_1(f)$. Transform $x = F^{1/2}$ and solve the resulting quadratic inequality. The result is straightforward with the back transformation.

Proof of Proposition 4: It is straightforward since:

$$\begin{aligned}\left(\frac{M_{2/3}(D)}{M_1(D)}\right)^{2/3} &= \left(\frac{(\sum_p D_p^{2/3}/r)^{3/2}}{\sum_p D_p/r}\right)^{2/3} \\ &= \frac{1/r}{(1/r)^{2/3}}\left(\frac{\sum_p D_p^{2/3}}{(\sum_p D_p)^{2/3}}\right) \\ &= \frac{1}{r^{1/3}}\sum_p\left(\frac{D_p}{\sum D_p}\right)^{2/3}.\end{aligned}$$

Proof of Lemma 1:

$$\begin{aligned}\text{TC(PF)} &= b\int_{\mathcal{M}}\sum_p D_p(x,y)dxdy + \tilde{c}F^{1/3}\int_{\mathcal{M}}\sum_p D_p(x,y)^{2/3}dxdy \\ \text{TC(FMF)} &= b_0\int_{\mathcal{M}}\sum_p D_p(x,y)dxdy + \tilde{c}F_0^{1/3}\int_{\mathcal{M}}\left[\sum_p D_p(x,y)\right]^{2/3}dxdy\end{aligned}$$

Note that, $M_1[D(x,y)] = \frac{1}{r}\sum_p D_p(x,y)$ and $M_{2/3}[D(x,y)] = \frac{1}{r}\sum_p D_p(x,y)^{2/3}$. The lemma could be proven easily by regrouping the elements.

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